Problem II.4 ... parallel collision

6 points; průměr 2,59; řešilo 99 studentů

The FYKOS-bird watches in their inertial frame of reference as two point masses move around them on parallel trajectories with constant non-relativistic velocities. They think whether these trajectories could intersect for some other inertial observer. If so, is it possible that the two point masses in question could collide at this intersection given the right initial conditions? Is this consistent with the fact that they are moving in parallel according to the FYKOS-bird?

Marek J. loves collisions.

The motion of two point masses can be described by using vectors. Since it is a uniform linear motion, the complete information is given to us by their velocities, which we denote as $\mathbf{v}_1$ and $\mathbf{v}_2$. In the FYKOS-bird's reference frame, they are parallel, so their vector product is **zero**. Transformation of the velocities into another inertial frame reduces to just subtracting the velocity of this new frame ($\mathbf{v}_n$) from the given velocities

$$\mathbf{v}_{1n} = \mathbf{v}_1 - \mathbf{v}_n \,,$$
$$\mathbf{v}_{2n} = \mathbf{v}_2 - \mathbf{v}_n \,.$$

And for them to intersect, the condition $\mathbf{v}_{1n} \times \mathbf{v}_{2n} \neq \mathbf{0}$ must hold, and therefore the following equation

$$(\mathbf{v}_1 - \mathbf{v}_n) \times (\mathbf{v}_2 - \mathbf{v}_n) \neq \mathbf{0} \,,$$

can be rewritten by using properties of the vector product, the condition $\mathbf{v}_1 \times \mathbf{v}_2 = \mathbf{0}$, and the identity $\mathbf{v}_n \times \mathbf{v}_n = \mathbf{0}$ as

$$(\mathbf{v}_2 - \mathbf{v}_1) \times \mathbf{v}_n \neq \mathbf{0} \,. \tag{1}$$

Relation (1) holds for all $\mathbf{v}_n$ that are not parallel to the trajectories $\mathbf{v}_1$ and $\mathbf{v}_2$, suppose that $\mathbf{v}_1 \neq \mathbf{v}_2$. In the case of equality, the trajectories will always be parallel. However, note that the condition given by equation (1) is necessary but not sufficient (consider the motion of the point masses parallel to the x-axis of the Cartesian system and the transformation given by the velocity with only the z-component. In 3D, trajectories can also be skew lines). We can still see that there are infinitely many observers for whom the given trajectories intersect! (To demonstrate, let us transform the earlier mentioned example with zero z-component. Then any velocity with non-zero components in the x- and y-direction transforms the trajectories to intersecting ones.)

However, is it actually possible for the two point masses to collide? That would require their mutual distance to be zero at some point in time. Let us describe the trajectories of the point masses mathematically. Let us choose the position of the first point mass at (some) time t_0 as the origin of the coordinate system. Its trajectory is then given by $\mathbf{d}_1 = \mathbf{v}_1 t$. Let us say that the FYKOS-bird saw them at the time t_0 and at the mutual distance $\mathbf{s}$. This means that the trajectory of the second point is described by the expression $\mathbf{d}_2 = \mathbf{v}_2 t + \mathbf{s}$. The transformation to another inertial frame is accomplished by substituting the transformed velocities $\mathbf{v}_{1n}$ and $\mathbf{v}_{2n}$ that are mentioned above. Thus, for the collision to happen

$$\mathbf{d}_{1n} - \mathbf{d}_{2n} = \mathbf{0} \,,$$

what gives us

$$(\mathbf{v}_1 - \mathbf{v}_2)\, t = \mathbf{s} \,,$$

which is only valid if $\mathbf{s}$ is parallel to the velocities in the FYKOS-bird's reference frame. In other words, the point masses in the question follow the same trajectory, and only overtaking is possible (for some magnitude of their velocities). Due to different trajectories, the collision of point masses is not possible even in different inertial frames.

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