Problem II.P ... planetary atmosphere

10 points; průměr 3,73; řešilo 90 studentů

What parameters does a planet need to have to keep its atmosphere comparable to the Earth? What conditions are essential for the planet to gain such an atmosphere?

Karel has remembered a task.

We owe the existence of the Earth's atmosphere to various physical processes. Others are responsible for its protection. However, as we shall see, the chemical and biological processes are also important in addition to the physical ones. Before we begin our discussion of these processes, we need to clarify our interpretation of the question from the assignment – What does it mean that an atmosphere of a planet is comparable to the Earth's atmosphere? Supposing that we are interested in the habitability of such a body, we want a planet with an atmosphere with the following properties:

- atmospheric pressure $p_\mathrm{a} \approx 100\,\mathrm{kPa}$,
- balanced surface temperature $T \approx 300\,\mathrm{K}$,
- chemical composition with nitrogen and oxygen – in the first approximation given by a mean molecular weight $M_\mathrm{m} \approx 15\,\mathrm{g{\cdot}mol^{-1}}$.

These conditions will ensure that the atmosphere will have a similar thickness.

Thermal evaporation

The first process we will look at is the thermal motion of particles. The particle velocity of mass m in thermodynamic equilibrium at temperature T is given by Maxwell's velocity distribution

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 \mathrm{e}^{-\frac{mv^2}{kT}}\,,$$

which expresses what fraction of particles move at a velocity v, more precisely for a small interval of velocities of width $\mathrm{d}v$ we have a fraction $\mathrm{d}n = f(v)\,\mathrm{d}v$ of particles. The characteristic feature of this distribution is that even for arbitrarily large velocities we can find some particles that reach them. If a particle has a velocity greater than the escape velocity and does not collide with anything on its path, it will irreversibly escape from the atmosphere. The escape velocity is given by the formula

$$v_\mathrm{u} = \sqrt{\frac{2GM}{R}}\,,$$

where M is the mass of the planet, R is its radius, and G is the gravitational constant. We have an atmosphere of mass M_atm, which has a pressure p at the surface and a temperature T that is relatively invariant with height. The characteristic height H of such an atmosphere can be estimated from the relation for the hydrostatic pressure and the equation of state for an ideal gas

$$p = \rho H g = H\frac{pM_\mathrm{m}}{R_\mathrm{g}T}\frac{GM}{R^2}\,, \qquad H = \frac{R^2}{GM}\frac{R_\mathrm{g}T}{M_\mathrm{m}}\,,$$

where R_g is the molar gas constant and M_m the mean molar mass of the gas. Thus, if we look at the layer of the atmosphere where the particles begin to be sufficiently sparse, we get the following formula for the mass flow rate

$$\frac{\Delta M_\mathrm{atm}}{\Delta t} \approx M_\mathrm{atm} n(v > v_\mathrm{u}) \frac{v}{H} \approx M_\mathrm{atm} \frac{v}{H} \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi \int_{v_\mathrm{u}}^{\infty} v^2 \mathrm{e}^{-\frac{mv^2}{kT}} \,,$$

We are mainly interested in very fast particles because the normal speed of a particle in Earth's atmosphere is a few hundred $\mathrm{m\cdot s^{-1}}$, while the escape velocity is about ten $\mathrm{km\cdot s^{-1}}$, i.e. $mv_\mathrm{u}^2 \gg kT$. In addition, exponential decay dominates in this case and so we can estimate v^2 before the exponential as vv_u

$$\begin{aligned}\frac{\Delta M_\mathrm{atm}}{\Delta t} &\approx M_\mathrm{atm} n(v > v_\mathrm{u}) \frac{v}{H} \approx \\ &\approx \frac{M_\mathrm{atm} v_\mathrm{u}}{H} \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi \int_{v_\mathrm{u}}^{\infty} v v_\mathrm{u} \mathrm{e}^{-\frac{mv^2}{kT}} \approx \\ &\approx \frac{M_\mathrm{atm} v_\mathrm{u}}{H} \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 2\pi v_\mathrm{u} \frac{kT}{m} \mathrm{e}^{-\frac{mv_\mathrm{u}^2}{kT}} \,.\end{aligned}$$

From this relation, we see that the lightest particles escape the atmosphere first. Let us consider the state of the atmosphere after a lapse of time on a scale of a few billion years $\Delta t = 10^{17}\,\mathrm{s}$. Over this time, we want to keep the atmosphere of nitrogen, but we want to get rid of the entire atmosphere of hydrogen and helium. Therefore, we substitute $\Delta M_\mathrm{atm} = M_\mathrm{atm}$ a $m \approx 10 m_\mathrm{u}$ (where m_u is the atomic mass unit) into the previous relation

$$\frac{v_\mathrm{u}^2}{H} \left(\frac{m}{2\pi k\, T}\right)^{\frac{1}{2}} \mathrm{e}^{-\frac{mv_\mathrm{u}^2}{kT}} = \frac{1}{\Delta t} \approx 10^{-17}\,\mathrm{s}^{-1} \,.$$

The expression is again dominated by the exponential element, so we insert values for the Earth as a suitable order of magnitude estimate into the terms before it. This gives a value of about $3\,\mathrm{s}^{-1}$. Thus, we need the exponential to be 10^{-17} and argument of exponential to be about $-18 \cdot \ln 10 \approx -39$. If we want a reasonable surface temperature ($300\,\mathrm{K}$) we get a value for the escape velocity $v_u = \sqrt{40kT/m} \approx 3.1\,\mathrm{km\cdot s^{-1}}$, which is an estimated minimal value of the escape velocity required to maintain an atmosphere containing heavier elements. We see that the Earth, Mars and Venus satisfy this condition, whereas the Moon does not. If we substitute the mass from the relation for density (for $\rho = 5\,\mathrm{g\cdot cm^{-3}}$) we get the condition for the minimal radius

$$R > \frac{v_\mathrm{u}}{\sqrt{\frac{8G\rho\pi}{3}}} \approx 1\,900\,\mathrm{km} \,.$$

It should not be forgotten that the sustainability of the atmosphere can also be affected by the fast rotation of the body – if the Earth had a rotation period of 1 hour, its equator would rotate with the magnitude of the escape velocity. Another possible influence is the gravitational action of another object either during a random close approach, where the atmosphere may "flow" between bodies and into the surrounding space, or by tidal action in the case of a satellite, or vice versa, the effect of the parent planet on a satellite.

The influence of the star

To maintain a comfortable surface temperature, it is necessary to be at the right distance from the parent star in the so-called habitable zone. This question will not be fully addressed, but we refer you to.[1] Let us rather focus on the adverse effects that the star exerts on the atmosphere. One of them is the photodissociation of molecules. In the atmosphere, high-energy (usually ultraviolet) photons can split molecules into atoms they are composed of. This will reduce the mass of the particles and a vast amount of energy will be released, while their velocities in equilibrium will be higher and they will escape more easily from the Earth's gravitational field.

However, much more dangerous are the most energetic particles, which come to the planet from the star– as stellar wind particles. These particles, mostly protons, flow towards Earth at speeds typically around $300\,\mathrm{km\cdot s^{-1}}$. If these very energetic particles reached the particles in the atmosphere, they would be accelerated enough by collisions to reach the escape velocity. This is the process by which Mars, for example, lost its atmosphere. This erosion can only be prevented by the presence of the planet's magnetic field. Estimation of the required magnitude of the magnetic field can be obtained from the relation for the motion of a charged particle in a magnetic field. The protons in the earth's magnetic field will move along spiral trajectories (helical trajectories) wound on magnetic field lines, while the radius of the spiral is given by the well-known relation for Larmor radius. We modify it to the form for the magnitude of the magnetic induction

$$B = \frac{mv}{qR_\mathrm{L}} = 5 \cdot 10^{-11}\,\mathrm{T}\,,$$

where we have substituted a rather arbitrary value of ten radii of the Earth for R_L – the range of the Earth's magnetosphere in the direction of the Sun in which we want the particle to be able to "rotate". This is therefore a very rough estimate of the minimal value. For a more accurate description, we would have to consider the effect of the pressure of the charged particles on the magnetic field, which is beyond the scope of this problem. The result of such a view, when comparing the energy density of the particles and the energy density of the magnetic field is the so-called Chapman-Ferrar distance[2]

$$R_{C-F} = R \left(\frac{B^2}{\mu_0 \rho v^2}\right)^{\frac{1}{6}}\,,$$

where we require that $R_\text{C-F} > R$ for mass ejections from the star as well.[3] Thus, we get condition $B > \sqrt{\mu_0 \rho v^2} \approx 5 \cdot 10^{-7}\,\mathrm{T}$, which is a much (by four orders of magnitude) stronger field than in the first case (in the previous case we considered a homogeneous field, which decreases with the third power of distance from the surface), but still more than two orders of magnitude weaker than the Earth's field. A strong magnetosphere can also prevent the atmosphere from being "blown away" after a mass eruption from the surface of a star. The high-energy particles are then trapped on spiral trajectories along magnetic field lines. Some of these particles can enter the atmosphere, where they cause auroras or the so-called Van Allen radiation belts at greater distances from Earth. However, from polar regions particles can still escape, as the field

[1] `https://www.astro.princeton.edu/~strauss/FRS113/writeup3/`, `https://en.wikipedia.org/wiki/Circumstellar_habitable_zone`

[2] `http://sun.stanford.edu/~sasha/PHYS780/SOLAR_PHYSICS/L23/Lecture_23_PHYS780.pdf`

[3] ρ, v are the parameters of the plasma impinging on the magnetic field, which for Earth reach a maximum of about $1\,000\,\mathrm{km\cdot s^{-1}}$ and $m_\mathrm{p} \cdot 100\,\mathrm{cm^{-3}}$. The actual values can be found e.g. at `https://www.swpc.noaa.gov/products/real-time-solar-wind`

lines are mainly in the direction towards the Earth open. This phenomenon is called a polar wind or plasma fountain.

The Magnetic Dynamo

Our planet needs to somehow generate this field. When large bodies are created, a melting process occurs during the initial collisions and subsequent gaining of mass as smaller objects bombard it. The magnetic field of a planet cannot be a field of permanent magnets– the material would certainly be above its Curie temperature – but must be generated by a process of a magnetic dynamo in a liquid conducting core of the planet. For Earth-like planets, the core consists of molten metals like nickel and iron. The planet must keep this core liquid for a sufficiently long time (after solidification of the core, the atmosphere will vanish rather quickly, as in the case of Mars), which again puts conditions on the size of the planet. The heat capacity increases with the third power of the radius, while the energy conducted to the surface and radiated by the surface increase with the second power. If we use the conduction equation of heat we get

$$Mc\frac{\Delta T}{\Delta t} \approx -k\frac{T}{R}4\pi R^2 \,,$$

where c is the specific heat capacity, k the thermal conductivity, and T the temperature of the planet. If we assume exponential cooling according to $T = T_0 \exp(-t/\tau)$ we get

$$\tau = \frac{Mc}{k4\pi R} \approx \frac{c\rho R^2}{3k} \,.$$

If we insert the values for iron, we get for $\tau = 10^9$ years radius $R = 1\,500\,\mathrm{km}$. This is just a rough estimate since this heat must still reach the surface through the less conductive silicate layers (the Earth's crust) and radiate from the surface, to which extra heat is supplied by the Sun. Another process that can keep the planet's interior fluid is tidal action, as in the case of Mercury (the Sun's action), or Io (Jupiter's moon).

The formation of atmospheres

To understand the formation of the atmospheres of planets, as well as that of the Earth, we must first understand the formation of the planets themselves. The planets are formed from gas-dust accretion disk which is the remnant of the formation of the central star. Depending on the distance from the star, the gas gradually condenses into particles– in the direction from the star, first metals, later silicates, water ice, and at great distances, ice of methane or ammonia. These particles subsequently accumulated by collisions into asteroid-sized planetesimals. These collided further and with the help of their gravity formed protoplanets. Here comes an important moment for us– if such a planetesimal gained a mass greater than about ten Earth masses, it has acquired the ability to gain the gas of a colloidal nebula with its gravity, further increasing its mass. This process created the atmospheres of gas planets which make up most of their mass [4]. If we want to get an Earth-like atmosphere, our protoplanet must have a radius within about

$$R < R_\mathrm{z}\left(\frac{10M_\mathrm{z}}{M_\mathrm{z}}\right)^{1/3} \approx 14\,000\,\mathrm{km}\,,$$

[4] For those interested, this process is described in scientific papers https://www.sciencedirect.com/science/article/abs/pii/0019103586901223, or https://academic.oup.com/mnras/article/405/2/1227/1184435.

if we assume that the densities of the two bodies are equal. For the evolution of (proto)planets, their migration in the system after their formation is also important. Because of the mutual gravitational interactions between multiple planets, the resulting orbits may differ significantly from those on which the planets formed; some planets may even fall onto the parent star or escape the system altogether.

The development of an Earth-like atmosphere usually begins on a body melted by collisions, whose primary atmosphere consists of molten minerals and volcanic gases such as methane. By later collisions with smaller bodies, however, water came to Earth from distant parts. This water as steam then contributed to the composition of the atmosphere. Gradual cooling ensured the solidification of the surface and later a water ocean could precipitate out of the atmosphere. Due to volcanic activity, this "second" atmosphere is composed of carbon dioxide and nitrogen, such as on Venus. Some of the carbon dioxide molecules may dissolve in the oceans and subsequently store in carbonates, and thus eliminate from the atmosphere. However, another phenomenon that has little to do with astronomy or geophysics was significant for the Earth's atmosphere – the origin of life. The first photosynthesizing organisms – cyanobacteria – were able to produce oxygen from carbon dioxide, to enrich the atmosphere. Oxygen can also be produced in the higher layers of the atmosphere by the photodissociation of steam, but this process is too slow compared to the amount of oxygen needed to oxidize minerals at the surface. It is these first organisms that we can be grateful for today's form of the Earth's atmosphere in which we, humans, can live.

Summary

The planet must be big enough to hold the atmosphere. As a result of the thermal motion of molecules, objects with a radius of less than a few thousand kilometers will lose the atmosphere on a scale of billions of years. Further, such a planet must possess a magnetic field shielding the particle stream from the parent star. Finally, the planet cannot be too big, otherwise, it would have started gaining gas from the surrounding nebula during its formation and turned into a gas giant. Thus, for the radius of the planet we have an estimation of $1\,500\,\mathrm{km} < R < 14\,000\,\mathrm{km}$. If we are interested in an atmosphere with oxygen, life would have to evolve on top of that.

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