Problem III.P ... by a flash

9 points; průměr 5,20; řešilo 46 studentů

What determines the width of a lightning channel in a thunderstorm? Create a quantitative model.
Karel stumbled upon a claim about the Sky Tower lightning rod.

Our goal is to estimate the diameter of the lightning channel according to various parameters. Before we proceed with the numerical calculations, let us first try to take a closer look at the lightning waveform and properties while qualitatively estimating which parameters might affect the thickness of the lightning channel.

Qualitative description

Lightning is an electrical discharge between a cloud and the ground. It results from the charge separation in a Cumulonimbus cloud, which results in inhomogeneous charge distribution in various parts of the cloud. The basic model we will work with throughout this solution is that the lower part of the cloud is negatively charged while the upper part is positively charged. There may be other smaller charge centers in the cloud, which we will neglect for simplicity.

There are several different types of lightning discharges. Approximately 90 % occurs within a cloud between its charge centers or between the charge centers of two adjacent clouds. However, these lightning flashes are not well studied and are not such a risk to humans on the ground. We will be concerned with the more well-known cloud-to-ground lightning, where we differentiate between two types. The more common (almost 90 %) are negative lightning bolts, which, as their name suggests, emanate from a negative charge center towards the ground. Rarer positive flashes of lightning originate from the positive charge center and occur, for example, when, due to high winds, this center is directly above the ground without being shielded by the negative charge center. These lightning flashes usually have a higher current and can occur in the high positive charge center, even when the storm is no longer directly over the site because of its origin. They may also coincide with other phenomena, such as supercharged lightning. However, for the sake of simplicity, we will only consider typical cloud-to-ground negative lightning.

Let's look at the different stages of the temporal evolution of lightning. Even if we have a cloud with separate charge centers, the electric potential between the cloud and the ground is about ten times smaller than the breakdown voltage of air. The origin of the initial discharge remains somewhat ambiguous, but it is likely that the breakdown voltage is locally reduced by showers of cosmic ray particles causing ionization and corona discharges at the tips of ice crystals. However, once the discharge reaches this initial phase, it develops a *lead discharge (leader)* that proceeds in steps towards the ground, ionizing the channel and distributing the negative charge from the cloud along its length, and sometimes branching off during this journey.

As the leader approaches the ground, it is met by a *return streamer* from the ground. When they connect, a so-called *return stroke* propagates upwards, balancing the potential difference and neutralizing the lightning channel. Then, a so-called *sustaining current* of at most a few kiloamperes can still flow through the ionized channel. If a sufficient amount of charge remains in the cloud, within a few tens of milliseconds, another leading discharge may take place through either the same channel or a nearby one, resulting in a sequential return stroke. This sequence can recur multiple times, with the average number of return discharges per flash ranging from three to five, although flashes featuring up to twenty return discharges have been recorded.

We will now consider what parameters might affect the thickness of the lightning channel. Since large currents flow through lightning, the thickness of the channel should undoubtedly be affected by the current. Its magnitude depends on several parameters, particularly the accumulated charge in the cloud, the position of the charge centers, and the initial conductivity of the air, which is affected by for example, pressure, temperature, and humidity. Furthermore, the lightning channel is influenced by the properties of the ground beneath it, such as the altitude, the conductivity of the subsoil, or the type of urban development.

Let's look more specifically at the physical phenomena that affect the thickness of the channel. In general, we assume that thickness is affected by influences that want to increase it and influences that want to decrease it, with the actual thickness being that at which these influences balance out. So, what factors can contribute to the widening of the lightning channel? First, the greater the thickness of the channel, the lower its electrical resistance, allowing for smoother passage of current. Also, thermal expansion and diffusion of the heated channel into the surroundings will cause the channel to expand. What, on the other hand, limits the thickness of the channel? Most certainly the lack of electrical charge to ionize a large area of space. Then, the magnetic field generated by the current flowing through the channel provides a pressure of several atmospheres in the channel.

Qualitative estimate

We will now attempt to examine the individual effects quantitatively in several different models. Due to the complexity of the initiation phase and the evolution of the leading discharge, we will not deal with these parts and take the charge density of the leading discharge as a parameter.

Tab. 1: Negative cloud-to-ground lightning parameters, data from Rakov and Uman(2003).

Quantity	leading discharge	return stroke	sustaining current
duration	35 ms	70 – 80 µs	100 ms
current	100 – 200 A	30 kA	100 – 200 A
channel temperature	10 000 K	30 000 K	–
total charge	5 C	–	–
transferred charge	—	5 C	10 – 20 C
electric potential	5 MV	–	–
channel thickness	–	1 – 2 cm	–

Model 1 – return stroke, given temperature and current In this first and simplest model, we will delve into the return stroke, where we estimate the channel thickness from the equality of the magnetic pressure that compresses the channel and the thermal pressure that expands it. We first express the magnetic pressure, which is generally of the form

$$p_b = \frac{B^2}{2\mu_0}\,, \tag{1}$$

where B is the magnetic flux density and μ_0 is the permeability of the vacuum. We will now try to express the magnetic field in terms of the current that passes through the channel. To achieve this, we will use Ampere's law

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \mu_0 I \ ,$$

from which we express the magnetic field on the surface of the channel with radius r, through which the current I passes, as

$$B = \frac{\mu_0 I}{2\pi r} \,. \tag{2}$$

Now, we will express the magnetic pressure with the combination of the equations (1), and (2)

$$p_b = \frac{I^2 \mu_0}{8\pi^2 r^2} \,.$$

We put it as equal to the thermal pressure $p_\mathrm{T} = nkT$, where n is the particle density, k is the Boltzmann constant, and T is the thermodynamic temperature in kelvins. We get the relation

$$\frac{I^2 \mu_0}{8\pi^2 r^2} = nkT \,,$$

from which we express the radius r as

$$r = \sqrt{\frac{I^2 \mu_0}{8\pi^2 nkT}} \,, \tag{3}$$

where after substituting the values of I, and T from table 1, and $n = 2.69 \cdot 10^{25}\,\mathrm{m^{-3}}$ calculated form the molar volume of ideal gas in standard conditions (1 013 hPa, 0 °C) we get $r \approx 1.13\,\mathrm{mm}$.

Using this reasoning, we derived the relation (3), which shows that the diameter of the lightning channel increases with current, which we would intuitively expect, but decreases with channel temperature, which is slightly counterintuitive. At the same time, we assume that current and temperature are not independent parameters but are coupled, which we will try to account for in the second model. The other thing we will try to correct in the following models is the resulting radius, which we found to be about an order of magnitude smaller than the tabulated values of 1.

Model 2 – equilibrium temperature As can be seen from the previous calculations, the channel during the return stroke is approximately an order of magnitude smaller in diameter than we would expect. Moreover, since the reverse discharge lasts only a brief moment, most of the transferred charge is due to the sustaining current. Since the durations of this current are relatively long (tens of milliseconds), we will consider the lightning channel to be in equilibrium. Let us compute its width under the assumption that all the heating caused by the passage of current I is converted into radiation of the channel as a blackbody. The radiated power is equal to

$$P_\mathrm{out} = 2\pi r l \sigma T^4 \,,$$

where σ is the Stefan-Boltzmann constant, and T is the thermodynamic temperature. The electric power is given by the Ohmic heating

$$P_\mathrm{in} = UI = RI^2 \,,$$

where the channel resistance is expressed by the conductivity Σ as $R = l/(\Sigma S)$, where l is the channel length, $S = \pi r^2$ is the area of the cross-section, and r is the desired radius. Channel conductivity Σ depends on the temperature and the plasma's ionization degree. For the lightning channel, we can estimate its value as $\Sigma = 3 \cdot 10^4\,\mathrm{S{\cdot}m^{-1}}$[1].

[1] Microphysics of Atmospheric Phenomena, Boris M. Smirnov, Springer Atmospheric Sciences, 2017

For the power of the passing current, we get the relation

$$P_{\text{in}} = \frac{I^2 l}{\pi r^2 \Sigma}\,.$$

By comparing the two powers, we get

$$\sigma T^4 = \frac{I^2}{2\pi^2 r^3 \Sigma}\,,$$

from which we can express the radius

$$r = \sqrt[3]{\frac{I^2}{2\sigma\pi^2\Sigma T^4}}\,.$$

Looking at the sustaining current from the table 1 and the channel temperature 10 000 K (we assume that the channel has cooled down a bit since the return stroke, but not too much to still have a fully ionized plasma for which the conductivity relation used applies), we get $r = 0.3\,\mathrm{mm}$, even less than in the previous model. Adding the values for the return stroke ($T = 30\,000\,\mathrm{K}$ and $I = 30\,\mathrm{kA}$) gives us a radius of $r = 3.2\,\mathrm{mm}$, which is larger than before. In reality, the channel is not in equilibrium in either of these situations. The channel is still heating up during the return stroke, resulting in a smaller radius. Conversely, the channel is cooling down during the sustaining current, leading to a larger radius. However, both calculated values are smaller than the expected radius.

The channel radius is proportional to the current, making it one of the most critical parameters. Additionally, for time durations in fractions of a second, diffusion begins to exert influence, a topic we will explore further in the subsequent model.

Model 3 – thermal diffusivity Since the sustaining current lasts for quite a long time and the temperature differences between the lightning channel and the surroundings are extreme, we try to consider another phenomenon – thermal diffusion. In the calculation, we will follow a method from the book Microphysics of Atmospheric Phenomena.[2] First, from Ohm's law, we express the lightning current as

$$I = \pi r^2 \Sigma E\,,$$

where E is the electric field. Next, we express the channel expansion using the thermal diffusion coefficient χ as

$$r^2 = 4\chi\tau\,,$$

where τ is the duration of the discharge. Combining these two, we get

$$\chi = \frac{I}{4\pi E\tau\Sigma}\,,$$

from which we can express the channel radius as

$$r = \sqrt{\frac{I}{\pi E\Sigma}}\,.$$

By substituting the values $E = 20\,\mathrm{kV{\cdot}m^{-1}}$, $\Sigma = 3\cdot 10^4\,\mathrm{S{\cdot}m^{-1}}$ and the relevant currents, we get $r = 4.0\,\mathrm{mm}$ for the return stroke, and $r = 0.2\,\mathrm{mm}$ for the sustaining current, which are very similar values to the previous model.

[2] Microphysics of Atmospheric Phenomena, Boris M. Smirnov, Springer Atmospheric Sciences, 2017

Conclusion

We have examined several simplified models for lightning discharge with different results for estimating its radius. Nevertheless, all were about an order of magnitude lower than we expected. The reason for this discrepancy is that the plasma within the observed channel is not homogeneous. The so-called *corona* discharge in the vicinity of the channel contributes a non-negligible amount to the measured thickness on the order of cm.

As expected, one of the largest influences on the channel thickness is the magnitude of the passing current. We counted with typical values of current in the order of tens of kA for the return stroke, but the maximal obtainable values are up to ten times higher.[3] Furthermore, our analysis assumed a constant current within a specific section of the lightning discharge. However, in reality, lightning is a highly dynamic process, with fluctuating currents and temperatures that prevent it from reaching equilibrium conditions. Additionally, lightning channels do not form straight lines between clouds and the ground; instead, they curve and branch in various directions, introducing additional resistance and capacitance to the channel.

Since the material inside the channel is plasma with a temperature higher than the temperature at the surface of the Sun, many of its thermal and electrical properties depend on the properties of the plasma, such as the degree of ionization, the collision frequency, and the effective cross-section of colliding particles inside the plasma. These factors influence the electrical conductivity and thermal capacity of the plasma, consequently impacting the temperature of the plasma, which may vary between electrons and ions. In addition, the rapid changes in currents generate waves in a broad spectrum of electromagnetic fields that carry away a non-negligible fraction of the energy, which allows us to detect lightning from distance and measure some of its properties (see, for example, the detection networks WWLLN[4] or EUCLID.[5]) An accurate model would therefore require very advanced plasma modeling. Nevertheless, a relatively simple description sufficed for an order-of-magnitude (millimeters to centimeters) estimate of the channel thickness.

Kateřina Rosická
`kacka@fykos.org`

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.

[3] `https://www.vaisala.com/sites/default/files/documents/Extreme%20Values%20of%20Lightning%20Parameters_A.%20Smorgonskiy%20et%20al.pdf`

[4] `https://wwlln.net/`

[5] `https://www.euclid.org/`