

Serial: Search for Quanta

This year we are going to take a look at quantum physics in the series. The details of mathematical formalism that quantum physics textbooks usually deal with will be relegated to the background. I don't want to reveal too much, so let yourself be surprised!

Today, we will talk about the foundations of quantum physics and its origin – it did not fall from the sky, but of course, it comes from observing the world around us. Now, it would be fair to look at and explain (some of) the experiments that led to the emergence of quantum mechanics as it is used today.

Historical Introduction to the Origin of Quantum Mechanics

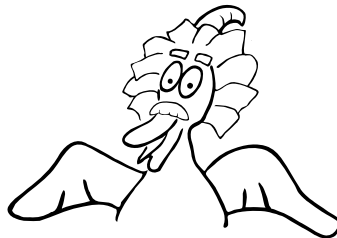
It was the end of the 19th century when it seemed that all the laws of physics were known and the physicists of that time had a theory describing all the physical interactions in the universe. This impression is beautifully summed up in a now legendary quote:

“There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.”

It is often attributed to Lord Kelvin, although it appears that he never uttered it.

At the time, no one knew how wrong this statement was. Of course, some indications showing the incompleteness of the physics of that time, which we now refer to as classical theory, were already known at that time. For example, according to classical theory, atoms should interact with all colors of light equally. But experiments showed that each atom prefers its favorite colors while not even noticing others. Furthermore, to explain some experiments, it was necessary to introduce the idea of the ether, which no one had ever seen (and later turned out to be redundant).

A little later, at the beginning of the 20th century, physicists began to investigate these inconsistencies, which resulted in a big blow to the classical view of the world. Two new theories emerged that fundamentally changed the understanding of the world at the time. Somewhat surprisingly, both were created by the same gentleman you've probably heard of before. . .



The first of these theories showed that the concept of space and time is not quite as rigid and universal as it was believed until then. Today it is known as the theory of relativity. In this series, however, we will be more interested in the second one – quantum theory. According to it, particles that are too small do not behave at all like ordinary objects that we know from the world around us, i.e., according to the laws formulated by the hit-in-the-head-by-an-apple genius more than two centuries earlier. The particles' behavior, on the other hand, is very strange according to our perception, and even stranger, but we are already getting a little ahead of it.

But let's return to the inconsistencies of experimental physics and classical theory. Already in the 19th century, it was known that bodies with a non-zero temperature emit electromagnetic radiation. The spectrum of such an object changes with its temperature. For temperatures typical around us, this radiation lies mainly in the infrared region. Therefore, unlike snakes or other vermin, human eyes cannot see it. If we want the object to glow even in the visible spectrum, it is enough to heat it to temperatures of the order of thousands of degrees Celsius (heat it to red, possibly white).

With the introduction of energy-saving light bulbs, this problem is quite topical. Manufacturers state the temperature of the light directly on the packaging, without the bulbs actually reaching such temperatures, so we don't have to worry about astronomically high temperatures. Instead, the temperature of a black body is indicated on the package, whose spectrum would appear to be the same as the spectrum of the given light source inside the package. (And this is despite the fact that, e.g., fluorescent lamps or many LED sources form white light from only a few specific wavelengths, compared to the relatively uniform distribution of a black body.) Paradoxically, sources labeled as warm have the lowest temperature because they tend to be colored red, which is perceived as a warm color. (Perhaps if the fire in the fireplace had 10000 °C it would be different.)

But let's go back to the absolute black body, and let's continue heating it. We start playing with plasma at a temperature of millions of degrees (I don't recommend it!), and we have bad luck again. Even in this case, our eyes will not see the radiation, because the emitted light will escape into the ultraviolet part of the spectrum. People working on the tokamak could tell you that the brightly glowing areas in it are, somewhat paradoxically, the coldest ones.

Anyway, the spectrum of absolute blackbody radiation was precisely measured long before the 20th century. At the same time, the Rayleigh-Jeans law was derived from thermodynamics and, in particular, statistical physics, i.e., fields that were developing rapidly at the end of the 19th century. This law gives very accurate predictions for long wavelengths. Unfortunately, it diverges for the opposite part of the spectrum (short wavelengths) and predicts infinite radiated energy in the ultraviolet region. I probably don't need to emphasize that such a result is highly problematic and could be described as non-physical. On the contrary, an empirical law for short wavelengths was known – Wien's displacement law, but it is not applicable for long wavelengths.

A perfect match between the theoretical prediction and the experimentally measured radiation of an absolute black body was obtained for the first time in 1901 by the German physicist Max Planck. He brought the previously unseen universality of the forecast by introducing a simple assumption. In his work, the body cannot release energy by radiation arbitrarily, but must do so in amounts of a precisely given size proportional to the frequency of the radiation. Amounts later came to be called quanta and gave the name to the entire field that later began to develop around this idea. Moreover, it was not a problem to determine the constant of this proportion-

ality h in the relation $E = h\nu$ from the known spectrum. Since then, this constant has been called Planck's, and it probably won't surprise you to see it again many times. Unfortunately, Planck did not have the right physical intuition, and he considered the whole quantum concept to be more of a mathematical trick than a new fundamental change in the understanding of physics.

When we shine ultraviolet light on the metal, we begin to eject electrons from it. But as soon as we reduce the frequency of the radiation below a certain limit, the electrons suddenly stop flying out, even if we increase the intensity of the low-frequency light as much as possible. In such a case, we would melt the metal before knocking electrons out of it (in fact, at temperatures around 1000°C , thermoemission occurs, but this is not very significant). The dependence of the photoeffect on the light frequency and the independence on the light intensity is very counterintuitive. Even Einstein was able to accurately determine the proportionality constant between the frequency of radiation and the energy of "amounts" from experimental data, and surprisingly (not so much for us) he received exactly the same value as Planck!

Unlike Planck, however, Einstein correctly concluded that this could not be a coincidence, so he elevated this mathematical trick to a fundamental physical law. Light must always propagate in quanta of precisely given energy. The energy cannot be arbitrary, but is given by the product of the radiation's frequency and Planck's constant. At the same time, he introduced the designation for these quanta, photons, which is still used today. Thus, he resurrected the corpuscular theory of light, which was considered outdated at the time.

More or less concurrently with this, there was an effort to explain the structure of atoms and molecules, which was characterized by the creation of theoretical models that were immediately refuted by new experimental data. At first, these were rather conjectures – John Dalton, at the beginning of the 19th century, considered them to be solid spheres; later, on the contrary, the mentioned Lord Kelvin expected them to be vortices in the ether. The first experimentally supported model came after 1897 when J. J. Thompson discovered electrons. He thus definitively proved that the atom has some internal structure, which he followed up in 1904 with his pudding model of the atom. He imagined the atom as negatively charged electrons floating in an evenly spread positive charge.

In the years 1908-1913, Ernest Rutherford and his collaborators conducted experiments with the scattering of alpha particles on a thin gold foil. He found that some of the particles were scattered and changed their direction very significantly; some even bounced back in the direction from which they came. This is incompatible with the Thompson model because, in a spread charge of the "pudding", the charged particles would scatter absolutely minimally. Rutherford interpreted the results (correctly!) as saying that the whole positive charge and mass in the atom are concentrated in the nucleus, which is almost point-like. This is how Rutherford's planetary model of the atom was created, where light electrons orbit around a heavy, almost point-like, positively charged nucleus. The energy of the electrons in such a model must necessarily depend on the radius of the path of the given electron. One of the biggest problems with this model was that it could not be stable according to the laws of classical electrodynamics. An electron orbiting the nucleus acts like a miniature antenna that emits electromagnetic radiation. Such radiation would remove energy from the system, leading to the electron falling into the nucleus within a fraction of a second.

Niels Bohr solved this minor deficiency with the seemingly simple assumption that electrons cannot move in atoms in arbitrary paths but only in specific orbits. Electrons can jump

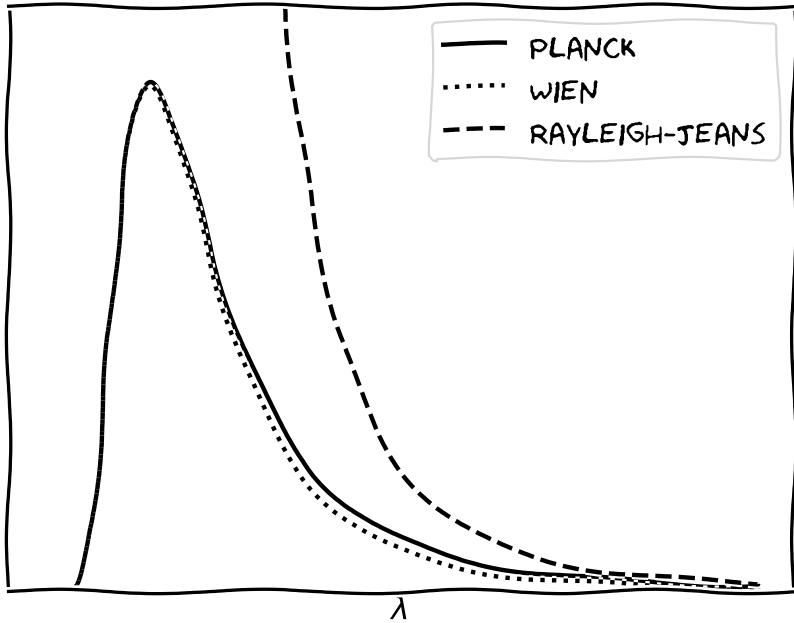


Fig. 1: Distribution of radiated power of an absolute black body by wavelength, as predicted by the three theories mentioned in the text. Note that although the Rayleigh-Jeans formula looks very different from Planck's, for very long wavelengths they give a similar result. Conversely, Wien's formula for long wavelengths predicts lower radiated power. Although this difference may seem small, the total power radiated over all wavelengths (area under the curve) is about 8% less, but it is still a better prediction than Rayleigh-Jeans.

freely between orbits or detach from the nucleus completely. At the same time, he explained a completely different phenomenon that we haven't mentioned yet. Atoms do not emit radiation continuously, but only at specific wavelengths – spectral lines.

Already at the beginning of the 19th century, Joseph von Fraunhofer (as well as William Hyde Wollaston before him) noticed that, when using a prism (or Fraunhofer using a diffraction grating he invented) to split sunlight into individual colors, black lines are present in the resulting color spectrum. These are interpreted in such a way that the given wavelengths are absent from the spectrum. Scientists were not lazy, and the empirical rules describing the behavior of spectral lines were created very quickly. Also, the wavelengths of the lines of the elements known at the time were measured.

The culmination came in 1868 when Norman Lockyer found a spectral line in the solar spectrum that did not correspond to any previously known element. Nevertheless, an unknown element, helium, was discovered and isolated even on Earth 20 years later.

For our story, however, the most interesting is the simplest spectrum, specifically the spectrum of the hydrogen atom. Hydrogen has only one electron, so its spectrum cannot be affected by the interaction between multiple electrons, which fundamentally affects the positions of the spectral lines. It was in this spectrum that Johann Jakob Balmer, who was somewhat obsessed with numerology, noticed that the wavelengths of the spectral lines of hydrogen could be expressed by a fairly simple relationship, which Johannes Rydberg then generalized to

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right),$$

where R is the Rydberg constant and n_1, n_2 are arbitrary natural numbers.

If we now return to Bohr's model of the atom, we find that it naturally includes these spectral lines. In such a model, the differences between the energies of the Bohr orbits would correspond to the spectral lines of the given atom. At the same time, the stability of the atoms is ensured (somewhat ad hoc) because the electron from the lowest orbit can no longer fall into the nucleus.

If we apply the Rydberg relation to the spectrum of the hydrogen atom, we find that it corresponds to jumps between levels at energies given by $\frac{hcR}{n^2}$, where n is the level number. From this, Bohr deduced that, similar to light quanta with Einstein and Planck before; we have a quantized quantity in this case as well. This time, it is the angular momentum of the electron, which takes on an integer multiple $\hbar = \frac{h}{2\pi}$ in the allowed orbits. Even though Bohr's model is basically true, it still cannot explain why this quantization occurs.

In 1924, Louis de Broglie came up with a revolutionary idea in his dissertation. If light behaves in some cases as waves and in other cases as particles, why couldn't entities that have been considered purely particles (for example, electrons) behave as waves in some cases? From the relations between the momentum and the energy of a photon of a specific wavelength, he deduced that the wavelength of an electron of momentum p should be

$$\lambda = \frac{h}{p}.$$

At the same time, he made the observation that the angular-momentum-quantization condition in Bohr's model corresponds to the fact that exactly an integer number of waves fit into a circular orbit (at the time when he considered the electron in this case as a wave). But part

of the scientific community was strongly skeptical of this idea. There is even a (rather unsubstantiated) story that de Broglie would not have defended his work if Einstein himself had not stood up for his dissertation before. On the other hand, another Nobel laureate, Max von Laue, is said to have said about this theory: “If that turns out to be true, I’ll quit physics.” (You can probably guess how it turned out: After de Broglie’s hypothesis was confirmed experimentally, he received the Nobel Prize in 1929 and Max von Laue devoted himself to physics until his death in 1960. Ironically, the experiment that proved de Broglie’s right was very similar to the one for which von Laue received the Nobel Prize.)

Around 1925, Werner Heisenberg began to formulate a rigorous theory of quantized electrons using the multiplication of non-commuting matrices, but since it is pretty complicated to understand, we will skip ahead to 1926. It was Erwin Schrödinger, who, based on de Broglie’s idea, formulated an equation that gives the de Broglie plane waves as a solution for the free electron’s motion. However, the general acceptance of this equation only came when it was shown that its solution for the hydrogen atom, i.e. an electron moving in the electrostatic potential of the nucleus, exactly corresponded to the experimentally measured spectrum.

This equation, which is still used in an almost unchanged form, will accompany us throughout the rest of the series. However, we will show this equation next time, including solutions for simple model systems.

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