Problem II.E ... the loudspeaker

12 points; průměr 9,05; řešilo 73 studentů

Measure the dependence of sound intensity emitted by your loudspeaker/mobile phone/computer on the distance from the source. Furthermore, determine the dependence of sound intensity on the settings of the output volume. Do not forget to fit the data.

Jarda cannot hear much in the back row.

Introduction and theoretical background

Sound intensity level is probably the most well-known acoustic quantity. Its unit is the decibel dB. Its long name suggests that this quantity is not as easy to understand as, for example, mass or length (at least in classical physics). It is defined as

$$L = 10 \cdot \log \frac{I}{I_0} \,,$$

where I is the sound intensity at the measured location, $I_0 = 10^{-12}\,\mathrm{W{\cdot}m^{-2}}$ is the intensity of the hearing threshold, and 10 appears here because the basic unit is the decibel, not just the bel. The intensity of a sound I is in turn the ratio of the sound power P to the area S through which it passes. Intensity and intensity level can thus be determined from each other using this relation.

The sound intensity level is therefore a dimensionless quantity. Moreover, it behaves logarithmically - if we double the sound intensity (for example, by adding an equally noisy source), L increases by approximately 3 dB. The logarithmic scale is chosen because the human ear perceives intensity logarithmically, and it can also operate over a range of about 12 orders of magnitude, so it is also useful for plotting noise on a graph.

Let's also think about how the dependence of the sound intensity level on the distance from the point source will behave. A point source is such a source that we can neglect its dimensions relative to the distance from it. Suppose that some power ΔP radiates from it to a solid angle $\Delta\Omega$. The area that this solid angle displaces on a sphere of radius r with centre at the source is $\Delta S = \Delta\Omega r^2$.[1] The intensity at distance r is thus

$$I = \frac{\Delta P}{\Delta S} = \frac{\Delta P}{\Delta\Omega}\frac{1}{r^2} \,.$$

The intensity therefore, not too surprisingly, decreases with the square of the distance. The sound intensity level will then vary as

$$L = 10 \cdot \log \frac{I}{I_0} = 10 \cdot \log \frac{\Delta P}{I_0 \Delta\Omega}\frac{1}{r^2} = 10 \cdot \left(\log \frac{\Delta P}{I_0\Delta\Omega} - 2\log r\right) \,.$$

It implies that it decreases as a constant minus the logarithm of the distance. But if we plot the distance on the x axis in logarithmic scale (i.e., instead of r we plot $\log r$), we should get a linear dependence with a coefficient of -20. We should note, however, that the notation in the form $\log r$ is not physically quite correct, and we should not divide the logarithm in the previous equation into the difference of two logarithms. For graphical illustration, however, the latter notation is better, which will allow us to plot the data points in a straight line and determine its slope.

Measurements will be made using a noise meter. If we don't have a more professional instrument, we will download an app on our mobile phone that allows this measurement.

[1] If the solid angle were equal to 4π, we get the surface area of the whole sphere

Measured values and results processing

To measure the dependence of sound intensity level on distance, we use a small loudspeaker that could represent a point source of sound. The measurements are made in a normal room, so we cannot completely avoid sound reflections from walls and furniture. We generate sound at a constant frequency of 440 Hz. For the actual measurement we use the *Sound Meter* application installed on the mobile phone.[2]

Tab. 1: Dependence of the sound intensity level on the distance from the source.

$\frac{r}{\mathrm{cm}}$	$\frac{L}{\mathrm{dB}}$	$\frac{r}{\mathrm{cm}}$	$\frac{L}{\mathrm{dB}}$
5	87	60	69
10	86	70	63
15	83	80	66
20	81	90	68
25	80	100	70
30	78	120	67
35	77	140	62
40	76	160	66
45	75	180	66
50	74	200	65

The measured dependence is displayed in the graph 1.

As mentioned in the theoretical introduction, we plot the horizontal axis on a logarithmic scale in the graph 2.

The data points in the graph were directly interleaved with the line.[3] The program itself determines the line that best fits the displayed points. In our case, this is after rounding the coefficients to the number of valid digits determined by the deviation

$$y = -\left(17 \pm 1\right) x + \left(101 \pm 3\right) \ .$$

The error in determining the coefficients will be shown by the program in which we searched for the coefficients.[4] However, we present the error values here mainly so that we can discuss the accuracy of the coefficients. According to the previous equation, the coefficient before x determines the rate of decrease of the sound intensity. For a point sound source, the intensity decreases as r^{-2}, for a linear source as r^{-1}. In our case, we get $r^{-1.7}$, closer to a point source as we would expect.

Next, we measured the dependence of the sound intensity level on the output volume setting, i.e. the so-called volume. This parameter will be denoted by v in our solution. We have measured at two distances ranging from 0 to 100 units (we denote by j). We used a laptop as the sound source.

We display the measured values in the graph 3. We also add the polynomials used to approximate the functions. Why do we intersperse the points with "random" polynomials? We

[2] Of course, other applications can be used, e.g. the well-known *Phyphox*

[3] For example, in *Excel* the function *trend line – linear*. The exact equation of the interleaved line can be obtained if we click on its display in the graph.

[4] In our case *Gnuplot*. Unfortunately, it is not easy to find this information in Excel.

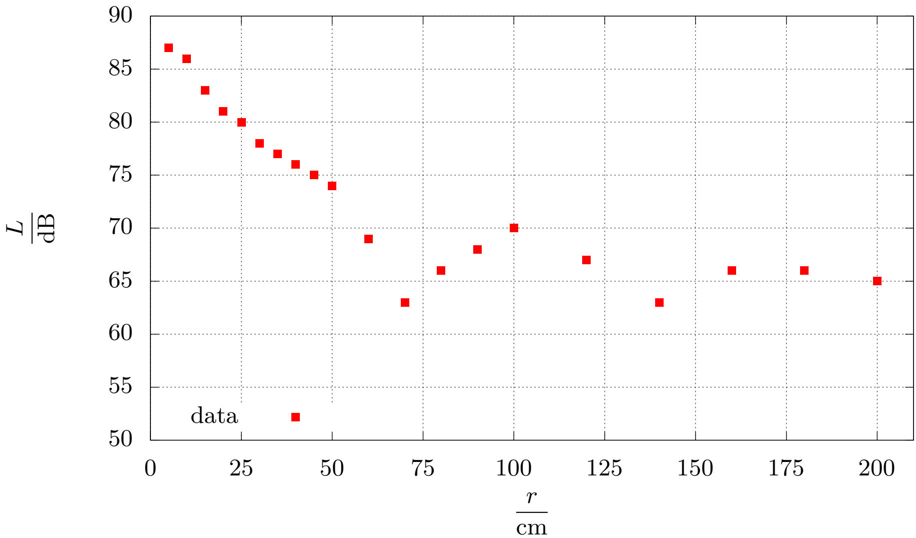

Fig. 1: Dependence of the sound intensity level on the distance from the source.

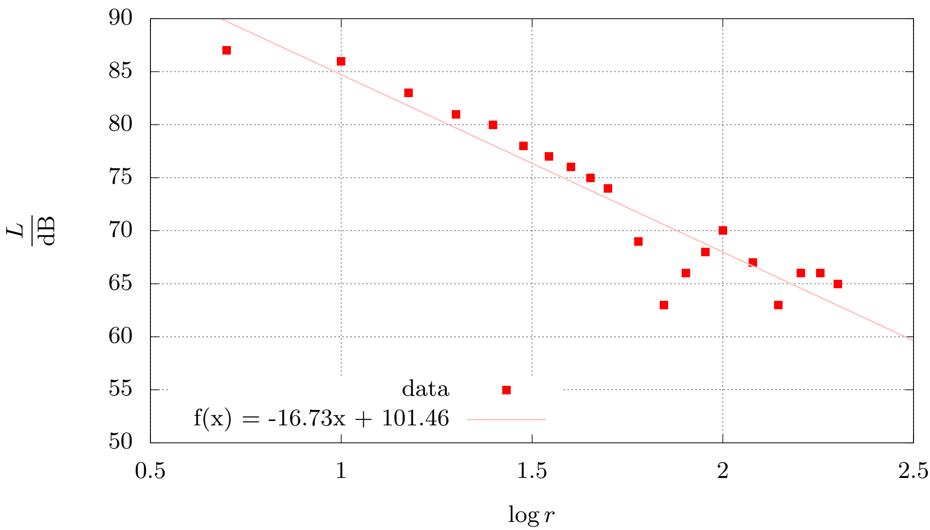

Fig. 2: Dependence of the sound intensity level on the logarithm of the distance from the source.

Tab. 2: Dependence of sound intensity level on volume. The index at L determines the distance D from the source in centimetres.

$\frac{v}{\mathrm{j}}$	$\frac{L_{15}}{\mathrm{dB}}$	$\frac{L_{50}}{\mathrm{dB}}$
0	35	34
4	40	35
10	48	40
16	52	45
22	57	49
28	60	52
34	63	55
40	66	58
46	68	60
52	70	62
58	71	63
64	73	65
70	74	66
76	75	67
82	77	69
88	78	70
94	79	71
100	80	72

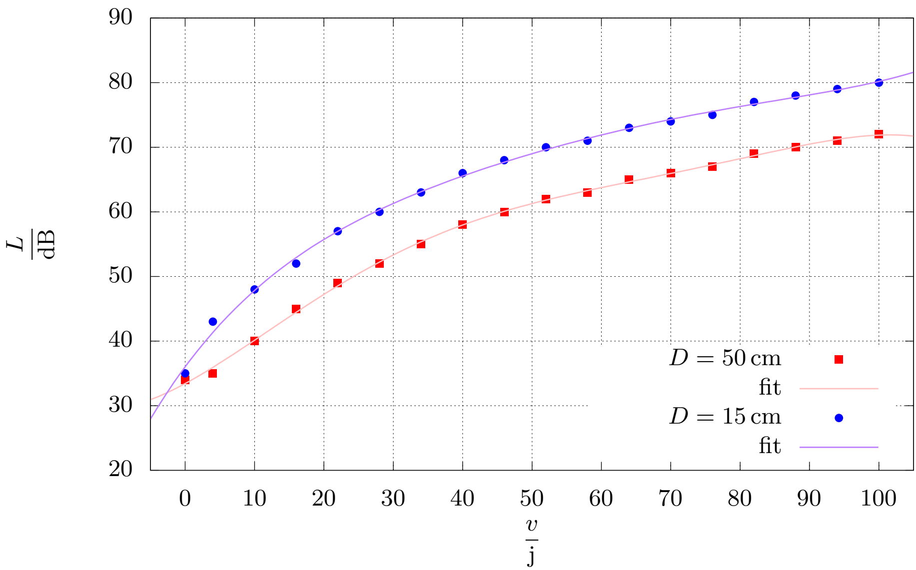

Fig. 3: Dependence of sound intensity level on volume.

have only measured a few data points. But if we were interested in retrospectively wondering what the sound intensity level would be at volume $v = 50\,\mathrm{j}$, for example, we can just plug in our polynomial and get a fairly accurate value. Moreover, we have no theoretical basis for interleaving the dependence with something else.

We interleaved the relation with a polynomial of degree 5 because we have quite a lot of data and lower degrees of polynomials would not be accurate enough for small or large values of v. For the dependence at $D = 15\,\mathrm{cm}$, the polynomial equation is

$$y = 1.2 \cdot 10^{-8} x^5 - 3.8 \cdot 10^{-6} x^4 + 4.6 \cdot 10^{-4} x^3 + 3.1 \cdot 10^{-2} x^2 + 1.5x + 36\,,$$

while for $D = 50\,\mathrm{cm}$

$$y = -2.3 \cdot 10^{-8} x^5 + 5.9 \cdot 10^{-6} x^4 - 5.3 \cdot 10^{-4} x^3 + 1.4 \cdot 10^{-2} x^2 + 5.8 \cdot 10^{-1} x + 33\,.$$

Discussion

In the theoretical introduction we considered the behaviour of the sound intensity level at different distances from the point source. We derived that the intensity decreases with the square of the distance. So if we plot the intensity level in decibels on the vertical axis, and the horizontal distance in logarithm, we get a line with a slope of $10 \cdot (-2) = -20$ (10 because we are working in tenths of a bel, and -2 is the power of the distance). In our case, we measured a slope of -17 ± 1, so the sound intensity decreases with distance as $r^{-(1.7\pm0.1)}$. Since this number is smaller than for a point source, it is clear that we cannot neglect the dimensions of the speaker. The relative error in determining this exponent is approximately 6 percent, so we can say that the data fit a linear relation and the determination of the coefficient is reasonably accurate.

The result may be inaccurate for several reasons. We only took measurements using a mobile phone app, which we are not sure will measure the same values as a professionally calibrated instrument. Better results could be obtained after calibrating the app, as is the case with the *Phyphox* app. At the same time, the app used does not indicate the measurement inaccuracy, so we cannot determine the measurement error.

The measurements took place in a normal room. Due to the presence of the walls and floor, the sound could be reflected and therefore affect the result.[5] On the other hand, the room was equipped with conventional furniture and carpet, so part of the sound was absorbed and not reflected towards the measuring phone.

But what is very clear from looking at the graph are two places where there is a significant reduction in intensity compared to the rest of the dependency. Their positions (approximately 70 cm and 140 cm) are not random, they are suspiciously close to multiples of the wavelength of the measured sound, which is

$$\lambda = n\frac{c}{f}\,, \tag{1}$$

where $c \doteq 340\,\mathrm{m\cdot s^{-1}}$ is the speed of sound propagation and $f = 440\,\mathrm{Hz}$ is the frequency used. For these values, 77 cm and 154 cm, which correspond approximately to the points mentioned. Thus, as a result of the experimental setup, destructive interference occurs. We could be convinced of the correctness of this theory if we measured the positions of the points where the intensity decreases significantly for other frequencies. It would turn out that this dependence follows approximately the equation (1).

[5] We can imagine it like putting a lamp next to a mirror – we also see it twice and more light comes to us

When measuring the intensity level versus volume, we can notice that the graph does not grow linearly. At lower values the increase is steeper, at higher values it is slower. So if we increase our volume, the higher we are, the less effective it is. However, the measured dependence probably depends on the particular instrument and software.

Conclusion

We measured the dependence of the sound intensity level on the distance from the source and plotted it on a graph. We found that the sound intensity decreases with distance approximately as $r^{-(1.7\pm0.1)}$. The speaker thus behaves approximately like a point source.

We measured the sound intensity level dependence at two distances. At each of the two distances from the source (laptop), we determined a fifth-degree polynomial that described the distance well. It is clear that the sound intensity level does not grow linearly with volume, but grows more slowly for larger values than for smaller ones.

Jaroslav Herman
`jardah@fykos.org`

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