Problem V.P ... tense tree

10 points; průměr 6,22; řešilo 51 studentů

Describe as many natural influences as possible that cause uprooting/severe damage to a lone tree in a meadow. Try to analyze one of them qualitatively as best as you can. What is the difference between a broadleaved tree and a conifer?
Bonus Discuss some of the influences quantitatively.

Danka watched the trees bend in the wind.

First, we will describe the structure of a broadleaf tree and explain how natural influences affect it. At the end of the text, we will compare the characteristics of broad-leaved and coniferous trees.

The first third of a broadleaf tree's height above the ground is made up of the trunk, which then branches out. Trees growing in meadows have plenty of space, light, and nutrients, as they are not competing with other trees, and so their crown is relatively large and densely branched.

Below the ground, we can observe the root system. Its structure varies considerably between species and several types of root systems exist. However, the division into shallow and deep-rooted species is sufficient for our purposes. We can assume a softer substrate in meadows, so there should be no flattening of the root system due to obstacles such as rocks. on the contrary, the less nutritious the soil is, the more robust the root system is (the tree must cover a larger volume of soil to obtain the necessary nutrients). The shape of the root system, and therefore the physical model of a tree in a meadow, can vary greatly depending on the tree species or the habitat.

A solitary tree in the middle of a meadow is exposed to several natural phenomena that can damage it. We will now list and briefly describe at least some of them. We will focus on describing the visible major damage, i.e., we will leave out effects such as drought and lack of nutrients causing the tree to die.

High winds

A broad-leaved tree has a high center of gravity due to its massive crown; especially in a densely-leaved tree, the crown represents a large surface area. In the event of strong winds, a large force is exerted on the top of the tree (in storms, trees are often very visibly bent in the direction of the wind). It can be described by Newton's drag formula

$$F = \frac{1}{2} C \rho S v^2 , \tag{1}$$

where S is the cross-section of the branch perpendicular to the direction of airflow, ρ is the density of the air, and v is the magnitude of its velocity. C is an experimentally determined coefficient of drag of the branch.

Using the lever principle, we add the individual contributions of the torque of the wind forces on the branch.

We can express it using the integral

$$M = \int_0^l \frac{1}{2} C \rho R(x) v^2(x) x \mathrm{d}x .$$

The torque exerted by the wind on a branch of length l is calculated as the product of the force at point x and the distance of this point from the origin of the coordinate system (also x), where we have placed the locus of the torque. We have approximated the cross-sectional area of the branch in the direction of the wind from the equation (1) by a rectangle with a length of side $R(x)$, which may be the radius of the branch at that point, and the other side has been chosen to be infinitesimally small, i.e., dx.

The force is thus transmitted along the branches through the trunk to the root system, where the energy obtained in this way is dissipated, i.e., converted into heat.

The torque of the wind is balanced by the torque of the compressive forces inside the branch. The stress depends on the flexural modulus (specific for each material), the force applied, and the shape of the branch cross-section.

$$M = \left| \int_S \sigma y \, dS \right| ,$$

where y is the distance of the surface element from the center of the cross-section.

The stress can then be expressed as

$$\sigma = \frac{M}{W_0} .$$

W_0 is the section modulus in bending, which can be found in engineering tables. For a circular cross-section, it can be thought of as the quotient of the second moment of area (see equation (2)) and the circle's radius.

To give an idea, we attach a figure showing the dependence of the torque of the applied forces and the bending radius of the branch R

$$M = \frac{E}{R} I_a ,$$

where E is the flexural modulus ($\sim 10\,000\,\mathrm{MPa}$)[1] and R is the bending radius. I_a is the second moment of area describing how far from the branch center (y) an element of the cross-sectional area is located.

$$I_a = \int_S y^2 dS. \tag{2}$$

We can see that the greater the applied torque, the smaller the radius R and the more the branch bends (infinite radius corresponds to a straight branch), which is an intuitive result.[2] When a force is applied, the branch bends to equalize the torques. However, we must consider that the bent branch has a smaller effective cross-sectional area on which the wind is acting and thus, the Newtonian drag force will change.

If in a specific part of the tree, this stress exceeds the elastic limit ($\sim 100\,\mathrm{MPa}$), some part of the tree will snap. Breaking off a branch is more likely than uprooting the whole tree, as less force is required. While neither the crown nor the wind are homogeneous, some parts are under

[1] `https://www.drevenekonstrukce.cz/co-je-to-vaznik-krov-a-jak-drevene-vazniky-vybirat`

[2] more detailed description of the bending can be found here:
`https://physics.mff.cuni.cz/kfpp/skripta/kurz_fyziky_pro_DS/www/fyzika.html`

greater stress than others. Older dry branches are more prone to breakage as they no longer have the necessary flexibility. Young and thin branches have a higher proportion of bast than older ones. For this reason, young branches show better elasticity.

Broadleaf trees have relatively strong trunks (except for some species such as birch) and are less likely to break.

If the root system is disturbed or the substrate is very unstable, uprooting may occur.

The event that occurs is very dependent on the conditions. As can be seen by describing the physical model, the whole problem is very complex. We must resort to numerical solutions and mathematical modeling methods to obtain specific numbers.[3]

Lightning

Natural electrical discharges - lightning - may occur during a storm. Lightning always strikes the point in its vicinity with the highest electrical charge. Such places include the tops of buildings or trees. In the case of a thunderstorm over a meadow with one solitary tree, lightning is most likely to strike that tree. The lightning then travels through the body of the tree to the ground along the path with the least electrical resistance - hence the shortest path. In doing so, a large transfer of heat energy occurs. Depending on the size of the lightning bolt, i.e., the amount of energy it transfers to the tree, the bark may be damaged (the lightning leaves visible traces), split, or even set on fire. Similar damage can occur if the fire is transmitted to the tree by other means. Trees, like houses, can be protected from lightning by proper grounding.

The susceptibility of a tree to lightning depends on the height of the tree itself and its conductivity. Acacia trees, for example, have a very high susceptibility, while beech trees have a relatively low susceptibility.[4]

Landslide

Landslides can occur in areas with mining activity or in the aftermath of an earthquake. Often, a large part of a slope is swept away, and trees are torn down together with the soil. Partial damage to the root system or even complete uprooting may occur. This phenomenon is more often in deforested areas where the soil is not reinforced by a system of tree roots (again, solitary trees with flat root systems are more susceptible).

Animals, fungi, and other parasitic organisms

All the aforementioned phenomena have worse consequences for weak and previously damaged trees. Various parasitic fungi or small animals such as bark beetles, which disturb the homogeneity of the wood, contribute significantly to this damage. In addition, smaller trees can be damaged by larger animals that nibble the leaves and purposefully cut down smaller branches or trees to build dwellings (beaver). Damage can also occur naturally through human activity.

Even though many other influences can damage trees, these are not so common and will not be discussed further.

A tree that grows alone, unlike trees living in a forest, is better built and therefore more resilient as an individual. On the other hand, trees in the forest are more protected by the

[3] You can find a similar calculation and examples from the modeling of the situation in the document that was developed during the investigation of the collapse of the Christmas tree on Old Town Square in Prague in 2003: `https://akela.mendelu.cz/~xcepl/inobio/inovace/Biomechanika_stromu/2_vanocni_strom.pdf`

[4] `https://szkt.cz/wp-content/uploads/2020/01/blesky1.pdf`

surrounding trees. The likelihood of wind damage is thus significantly lower. However, various parasitic animals can spread easily in a forest as a consequence of their single-species composition (parasites are mostly food specialists).

Coniferous trees growing in the open have their branches very close to the ground, unlike broadleaved trees (with Scotch pine being an exception). At the same time, they have a shallower root system than broadleaf trees. That is why conifers are more likely to be uprooted. Most coniferous trees also have weaker trunks and therefore break more frequently.

Kateřina Charvátová
katerina.charvatova@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.