Problem V.S ... we are spending electricity

10 points; průměr 7,26; řešilo 31 studentů

1. *The aluminum smelter annually produces* 160 000 t *of aluminum, which is produced by electrolysis of alumina using a DC voltage of* $U = 4.3\,\mathrm{V}$*. Determine how many units of nuclear power plant with a net electrical output of* $W_0 = 500\,\mathrm{MW}$ *are equivalent to the energy consumed by the aluminum smelter.*

2. *A DC current of magnitude* I *is applied to a tangent galvanometer with* n *turnings of radius* R*. The compass needle is deflected by an angle* α *from the equilibrium position. Determine the relationship needed to calculate the flowing current.*

3. *Measuring the temperature* T *using a thermistor to determine its resistance* $r(T)$ *utilizes a Wheatstone bridge with three resistors of known values* R_1, R_2, R_3*. What voltage* $U(T)$ *do we measure on the voltmeter in the middle of the bridge?*

4. *In the second half of the last century, conventional electrical units were based on the values of the frequency of the cesium hyperfine transition* $\nu_{\mathrm{Cs}} = 9\,192\,631\,770\,\mathrm{Hz}$*, the von Klitzing constant* $R_{\mathrm{K}} = 25\,812.807\,\Omega$ *and the Josephson constant* $K_J = 483\,597.9 \cdot 10^9\,\mathrm{Wb}^{-1}$*. Determine the value of the coulomb* 1 C *using these constants.*

Dodo has dead batteries.

Problem 1

This series introduced the well-known Faraday's law of electrolysis in the form

$$M = \frac{QM_{\mathrm{Al}}}{F\nu}\,,$$

where M is the mass of aluminum produced over time $t = 1\,\mathrm{year}$, Q is the charge transferred during this time, $F \doteq 96\,500\,\mathrm{C{\cdot}mol^{-1}}$ is the Faraday constant and ν the number of electrons transferred per one reaction.

Atoms of aluminum in pure state have an oxidation number of zero, whereas the aluminum atoms in alumina (aluminum oxide) have an oxidation number of 3, as can be seen from the chemical formula of the compound

$$\mathrm{Al_2}^{\mathrm{III+}}\mathrm{O_3}^{\mathrm{II-}}\,.$$

The change in oxidation number represents the number of electrons involved in forming one aluminum atom. In this case, $\nu = 3$.

In the first equation, we put $M = 160\,000\,\mathrm{t}$ as the mass of aluminum produced in one year. To achieve that, a charge of $Q = 3MF/M_{\mathrm{m}}$ had to be transferred, which required work

$$W_Q = QU = \frac{3MF}{M_{\mathrm{Al}}}U \doteq 7.4 \cdot 10^{15}\,\mathrm{J}\,,$$

where we put $M_{\mathrm{Al}} \doteq 0.027\,\mathrm{kg{\cdot}mol^{-1}}$ as the molar mass of aluminum.

One unit of nuclear power plant with net electrical power $W_0 = 500\,\mathrm{MW}$ produces

$$W_{\mathrm{e}} = W_0 t = 15.8 \cdot 10^{15}\,\mathrm{J}$$

in one year of continuous operation, where $t = 1\,\mathrm{year} \doteq 31.5\,\mathrm{Ms}$.

By comparing these two results, we can infer that operating the aluminum smelter requires approximately

$$\frac{W_Q}{W_\mathrm{e}} \doteq 0.47$$

units of nuclear power plant. There is an aluminum smelter of similar size in Slovakia that indeed consumes several percent of the total electricity production in the country.

Problem 2

As explained in the series, the tangent galvanometer relies on the Earth's magnetic field and the field generated by the current in the coil – aligning the compass needle along the magnetic field lines. Initially, it is essential to calibrate the device so that the compass needle lies in the plane of the coils; in this position, it is aligned with the Earth's magnetic field lines. Let us denote the magnitude of this field by B_Z; on Earth, it is on the order of tens of T.

When we activate the current in the coils, it generates a magnetic field around the conductors. We aim to determine its direction and value in the middle of the coils, where the compass needle is positioned. Determining the direction is simple – since the coil is rotationally symmetric, the field must be perpendicular to the plane of the coils. It would violate the rotational symmetry in any other case.

We can easily calculate the magnitude of the field at the mentioned point using the Biot-Savart law. A detailed calculation is performed, for example, here[1]. We continue with the expression derived in the previous source

$$B_\mathrm{v} = \frac{\mu_0 I n}{2R}\,,$$

where μ_0 is vacuum permeability and the rest of the quantities are defined in the problem statement.

The compass needle now rotates in the direction of the resultant of the two fields. They are perpendicular to each other, so for the angle α, we have

$$\tan\alpha = \frac{B_\mathrm{v}}{B_\mathrm{Z}}\,,$$

which, after substituting, gives

$$I = \frac{2R}{n\mu_0} B_\mathrm{Z} \tan\alpha\,.$$

Problem 3

A voltmeter is an electric device with a very high internal resistance, permitting only a small current to flow through it. That minimizes the impact of the measurement on the rest of the circuit. We consider the current in the voltmeter to be very low in comparison with the current I_1 in the branch with resistors R_1 and R_3 and also with the current I_2 in the branch with the resistors R_2 and the thermistor $r(T)$. Thus, we can say that the current I_1 is the same in both resistors R_1 and R_3; similarly for the resistors in the other branch.

[1] `https://www.toppr.com/ask/question/a-using-biotsavarts-law-derive-an-expression-for-the-magnetic-field-at-the-centre-of-650eed/`

Since the voltage in both ends of both branches is U, then according to Ohm's law

$$(R_1 + R_3)\, I_1 = U \ = (R_2 + r(T))\, I_2 \,,$$

from which we get the ratio of the currents in both branches, as well as their magnitudes.

The voltage drop across the resistor R_1 in the first branch is $R_1 I_1$, and the voltage drop across the resistor R_2 in the second branch is $R_2 I_2$. The difference between these values corresponds to the voltage across the voltmeter. Thus, we get

$$U(T) = R_1 I_1 - R_2 I_2 = R_1 \frac{U}{R_1 + R_3} - R_2 \frac{U}{R_2 + r(T)} = U \left(\frac{1}{1 + \frac{R_3}{R_1}} - \frac{1}{1 + \frac{r(T)}{R_2}} \right) .$$

This is the voltage measured by the voltmeter. The sign depends on how we insert the voltmeter into the circuit and on the values of the specific resistors. For the voltage on the voltmeter to be zero and no current to flow through it, we obtain a well-known condition for the values of the resistors $r(T) = R_3 R_2 / R_1$.

Problem 4

The problem is a simple application of dimensional analysis. Firstly, let us convert the units of all three constants to SI units

$$\begin{aligned} [\nu_{\mathrm{Cs}}] &= \mathrm{Hz} = \mathrm{s}^{-1} \,, \\ [R_{\mathrm{K}}] &= \Omega = \mathrm{m}^2{\cdot}\mathrm{kg}{\cdot}\mathrm{s}^{-3}{\cdot}\mathrm{A}^{-2} \,, \\ [K_{\mathrm{J}}] &= \mathrm{Wb}^{-1} = \mathrm{A}\ {\cdot}\mathrm{s}^2{\cdot}\mathrm{m}^{-2}{\cdot}\mathrm{kg}^{-1} \,, \end{aligned}$$

We aim to combine these three expressions using multiplication and exponentiation to get a result with the unit A·s. At first glance, we can simply multiply R_{K} by K_{J} and then invert this product to obtain the desired result.

Thus, we have

$$Q = \frac{1}{R_{\mathrm{K}} K_{\mathrm{J}}} = \left(25\,812.807\,\Omega \cdot 483\,597.9 \cdot 10^9\,\mathrm{Wb}^{-1}\right)^{-1} = \frac{1}{12.483\,02} \cdot 10^{-18}\,\mathrm{C} \,,$$

and the answer is

$$1\,\mathrm{C} = 12.483\,02 \cdot 10^{18} \frac{1}{R_{\mathrm{K}} K_{\mathrm{J}}} \,.$$

For unit conversions of electromagnetic quantities, we recommend using the English Wikipedia. For instance, for the unit ohm, it is immediately found that it is the ratio of weber to coulomb.[2]

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[2] `https://en.wikipedia.org/wiki/Ohm`