Problem VI.S ... illuminating units

10 points; průměr 5,50; řešilo 30 studentů

1. *There is an isotropic (its properties depend on the direction) light source perpendicularly above the center of a table. The center of the table is illuminated by $E_1 = 500\,\mathrm{lx}$. The edge of the table is $R = 0.85\,\mathrm{m}$ from the center and is illuminated by $E_2 = 450\,\mathrm{lx}$. How far from the center of the table is the light source? What is its luminous intensity?*

2. *Measure the luminous intensity of your favorite lamp using one of the visual photometric methods mentioned in the series. Use a tea candle made of white paraffin wax as the unit of luminosity. Remember to describe your experimental setup and attach a photograph or a diagram. How accurate was your result?*

3. *Let's construct the "Earth" system of units using the values of the mean density of the Earth, the standard atmospheric pressure at sea level, the standard gravity of Earth, and the magnetic induction measured at the Earth's south magnetic pole $B_0 = 67\,\mu\mathrm{T}$. Calculate the values of second, meter, kilogram, and ampere in this system and find the values of the speed of light, Planck's constant, gravitational constant, and vacuum permeability in "Earth" units.*

Dodo's table light at dorms is out.

1. For an isotropic light source the illuminance E is independent on the direction from the source. However, it is still dependent on both the distance r and the angle between the normal to the illuminated surface and the direction to the light source α

$$E = \frac{I}{r^2}\cos\alpha\,,$$

where I is the luminous intensity of the source. If we denote the height of the light source from the table by h, using Pythagoras' theorem and the cosine definition, we get

$$E_1 = \frac{I}{h^2}\,,\quad E_2 = \frac{I}{h^2+R^2}\frac{h}{\sqrt{h^2+R^2}}\,.$$

We then divide one of those equations by the other

$$\frac{E_2}{E_1} = \frac{h^2}{h^2+R^2}\frac{h}{\sqrt{h^2+R^2}} = \left(\frac{h^2}{h^2+R^2}\right)^{\frac{3}{2}}$$

and finally, we get

$$h = \frac{R}{\sqrt{\left(\frac{E_1}{E_2}\right)^{\frac{2}{3}} - 1}}\,.$$

Substituting, we get $h \doteq 3.15\,\mathrm{m}$. Furthermore, we get the luminous intensity $I \doteq 5\,000\,\mathrm{cd}$ from the relation for E_1, and if the source is isotropic, the luminous flux is $\Phi = 4\pi I \doteq$ $\doteq 62\,\mathrm{klm}$ for this luminous intensity.

2. We measured the luminous intensity of the Xiaomi Redmi Note 11S' smartphone flash. We did not find the luminous intensity or luminous flux of this source in the manufacturer's documentation. As a reference source, we used a tea light candle made from white paraffin wax, which we let burn for a few minutes after lighting. We conducted the measurement in a dormitory room with no other light sources using the Bunsen method. A sheet of A5-sized paper with a semi-transparent wax spot about 8 mm in diameter served as a photometer. During the measurement, we kept the planes of both the phone and the sheet of paper perpendicular to the line connecting the light sources, ensuring that the spot also laid on this line. We conducted the measurement in an approximately horizontal plane.The distance $D = 197.4 \pm 0.3\,\mathrm{cm}$ between the light sources was determined with a tape measure. Next, we used a ruler to measure the distance d from the candle flame to the photometer. Although the measurement of length by itself had an accuracy of about 1 mm, determining the position when the spot was as bright as the surrounding paper was not easy – our determined distance $d = 17.2 \pm 0.8\,\mathrm{cm}$ includes this error. We also experimented with four candles, keeping the distance D constant and measuring $d = 33 \pm 2\,\mathrm{cm}$. The significant measurement error now accounts for the distance between the candles, as they cannot all be placed at the exact same spot.

   The luminous intensity of light I is determined using the formula

$$I = NI_0 \left(\frac{D - d}{d}\right)^2 \,,$$

   where I_0 is the luminous intensity of the candle and N is the number of used candles. After substituting into the formula, we get the luminous intensity $I = (110 \pm 10)I_0$ using one candle and $I = (99 \pm 15)I_0$ using four candles.

   We see that these values are consistent with one another. The bulk of the measurement error comes from the large relative error in the determination of d. In our case, two factors complicated the situation:

   - different color of light sources – the candle had a distinctly yellow light compared to the smartphone, which made it difficult to determine the position of the photometer in which the area is homogeneously bright. In addition, when comparing sources of different colors, it also depends on the effect of the transition between day and night vision;
   - significant imbalance in the intensity of the sources – the measurement was conducted close to the candle, and therefore, even a small absolute error had a large relative influence. Here, it is also important to use a spot that has a small angular size from the perspective of both sources (so that the angle between the surface and direction to the sources can be considered the same over the entire measurement area) and also a source with the angular size less than a few degrees from the perspective of the spot.

   In addition, systematic errors affected our measurement, mainly due to the influence of light scattering in the room. To counteract this effect, it is advisable to blacken the room where we take our measurements and the objects in it (including the experimenter) – that is, to paint them black or cover them with a black cloth.[1]

3. The first step is to find the values of the variables that we will fix to a unit value in SI units and convert them to their simplest form. We then get $\rho_0 = 5\,513\mathrm{kg{\cdot}m^{-3}}$ for the average density of the Earth, the atmospheric pressure at sea level $p_0 = 101\,325\mathrm{Pa} = 101\,325\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-2}}$, gravitational acceleration $g_0 = 9.806\,65\mathrm{m{\cdot}s^{-2}}$, and magnetic flux density $B_0 = 67 \cdot 10^{-6}\,\mathrm{kg{\cdot}s^{-2}{\cdot}A^{-1}}$. Consequently, we can determine the value of the kilogram, meter, second, and ampere using any of the methods for solving systems of equations. However, we can deviate from the general procedure due to known relations. From the relation for hydrostatic pressure, we get

$$h = \frac{p}{\rho g} \Rightarrow \frac{p_0}{\rho_0 g_0} = \frac{101\,325\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-2}}}{5\,513\mathrm{kg{\cdot}m^{-3}} \cdot 9.806\,65\mathrm{m{\cdot}s^{-2}}} = 1.874\,\mathrm{m} \Rightarrow 1\,\mathrm{m} = 0.5336\,\mathrm{p_0}{\cdot}\rho_0^{-1}{\cdot}\mathrm{g_0^{-1}}\,.$$

From this, we can easily determine the values of a kilogram and a second

$$1\,\mathrm{kg} = \frac{\rho_0 \cdot \mathrm{m}^3}{5\,513} = 27.56 \cdot 10^{-6}\,\mathrm{p_0^3}{\cdot}\rho_0^{-2}{\cdot}\mathrm{g_0^{-3}}\,,$$

$$1\,\mathrm{s} = \sqrt{\frac{9.806\,65\mathrm{m}}{g_0}} = 2.288\,\mathrm{p_0^{1/2}}{\cdot}\rho_0^{-1/2}{\cdot}\mathrm{g_0^{-1}}\,.$$

Then, we can determine the value of an ampere

$$1\,\mathrm{A} = 67 \cdot 10^{-6}\,\mathrm{kg{\cdot}s^{-2}{\cdot}B_0^{-1}} = 3.5 \cdot 10^{-10}\,\mathrm{p_0^2}{\cdot}\rho_0^{-1}{\cdot}\mathrm{g_0^{-1}{\cdot}B_0^{-1}}\,.$$

To determine the values of the natural constants from the problem statement, we only need to substitute the units of these quantities for the values of the SI unit we just determined. For the speed of light

$$c = 299\,792\,458\,\mathrm{m{\cdot}s^{-1}} = 6.993 \cdot 10^7\,\frac{\mathrm{p_0}{\cdot}\rho_0^{-1}{\cdot}\mathrm{g_0^{-1}}}{\mathrm{p_0^{1/2}}{\cdot}\rho_0^{-1/2}{\cdot}\mathrm{g_0^{-1}}} = 6.993 \cdot 10^7\,\mathrm{p_0^{1/2}}{\cdot}\rho_0^{-1/2}\,,$$

where the unit may remind us, for instance, the relation from Bernoulli's equation $(1/2)\rho v^2 = p$.

For Planck's constant, we have

$$h = 6.626\,070\,15 \cdot 10^{-34}\,\mathrm{kg{\cdot}m^2{\cdot}s^{-1}} = 2.273 \cdot 10^{-39}\,\mathrm{p_0^{9/2}{\cdot}g_0^{-4}}{\cdot}\rho_0^{-7/2}\,.$$

The gravitational constant has the value

$$G = 6.674\,30 \cdot 10^{-11}\,\mathrm{kg^{-1}{\cdot}m^3{\cdot}s^{-2}} = 7.031 \cdot 10^{-8}\,\mathrm{p_0^{-1}{\cdot}g_0^2}$$

and permittivity of vacuum

$$\varepsilon_0 = 8.854\,187\,818 \cdot 10^{-12}\,\mathrm{A^2{\cdot}kg^{-1}{\cdot}m^{-3}{\cdot}s^4} = 7.2 \cdot 10^{-24}\,\mathrm{B_0^{-2}}{\cdot}\rho_0^1\,.$$

[1] There is a problem in *Practical Course III – Optics* in which the directional dependence of the luminous intensity of a light bulb is measured using a digital photometer. The measurement takes place in a small black room, and during the measurement, the readings on the instrument can be influenced just by the position of the experimenter. That is also the case when wearing black jeans and a black FYKOS hoodie.

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